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PROSPECTIVE NOISE REDUCTION

BY DOMES ON PLANAR ARRAYS

I. Theoretical Estimates for Unilayers

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ABSTRACT

The reduction of direct turbulent boundary-layer noise in a planar array by a fluid unilayer interposed between the array and the flow is assessed from theoretical considerations. The layer is supposed composed of independent contiguous segments each of superficial dimensions large relative to the sound wave length in the interior. The noise due to the high-wavenumber convective component of excitation is estimated to be reduced relative to a similar flush-mounted array by more than 10 log α^{-1} + 10 log (a/R_o) db for any steering angle, where α is the pressure-sensitive fraction of total array area (array factor), $\underline{a}$ is a characteristic dimension of the face of each layer segment, and R_o is the radius (or equivalent dimension) of each element. Likewise, the noise due to the low-wavenumber component of excitation is estimated to be reduced by at least 10 log α^{-1} db provided, for the element spacing $D \simeq \lambda/2$ ($\lambda = 2\pi c/\omega$) characteristic of an active array, the layer thickness L satisfies $L \gtrsim 0.5D$. On the other hand, the noise due to a radiated acoustic field, including that associated with compressibility of the TBL, will not be appreciably changed relative to a signal. A reliable assessment of the overall efficacy of a dome thus depends critically on a correct determination of the relative contributions of the noise components for flush elements. The high-wavenumber contribution is thought to be at least as large as the low in the frequency range of concern, but uncertainty about the various contributions remains and is yet unresolved by measurements and present interpretation.

I. THEORETICAL ESTIMATES FOR UNILAYERS

"In Xanadu did Kubla Khan
A stately pleasure-dome decree ...."

Samuel Taylor Coleridge

1. Introduction

1.1 Unilayer-domed Array

A useful limiting type of dome to consider in the context of a planar array is a layer of fluid separating the sensing elements from the flow and itself separated from the flow only by an impedanceless membrane. To this simplified model are related the more realizable configurations of (a) a similar unilayer made of an elastic solid and (b) a fluid layer covered by a solid layer with nonvanishing impedance. We consider here a segmented fluid unilayer of thickness L whose n similar segments are mutually independent, having prescribed boundary conditions imposed on their interfacing edges, and together contiguously cover the entire area A_T of the array. Representing the face area of each segment by πa^2 (though each will typically be rectangular), we have $A_T = n(\pi a^2)$. (See Fig. 1.)

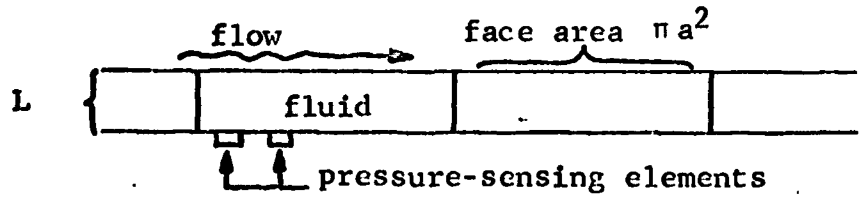

Figure 1. Edge View of Segmented Unilayer Dome

1.2 Wavenumber Spectrum of Exterior Noise Field

The effect of the unilayer on the noise-pressure field applied to its outer surface at a given frequency ω depends on the wave-number spectrum of this field. In the application of concern the noise pressure is distinguished as due to (1) direct turbulent boundary-layer (TBL) pressure fluctuations, (2) radiated acoustic field from sources many acoustic wave lengths distant, (3) acoustic field due to sources near the outer surface, e.g. bubbles, if any. In the case of a dome cover with nonvanishing impedance, we would add to (3) the pressure field associated with elastic waves in the cover. We may need to include an acoustic field due to direct vibrational excitation of the interfaces between segments if these include structural members.

1.3 Acoustic Noise; Roughness

For any given array steering angle, the effect of the dome on the radiated acoustic noise field (2) is substantially the same as its effect on a signal from this direction. Hence, a dome will not appreciably affect the acoustic noise-to-signal ratio. In the desired absence of near-field noise sources (3), then, a dome will alter the noise-to-signal ratio only by its effect on the TBL noise. It must be noted, however, that the TBL noise may depend on roughness, and hence that the maintainability of the dome surface vis-à-vis the bare array surface also is pertinent to the effect of the dome, as well as the influence of the dome on the noise due to the TBL on a smooth surface which we assume here.

1.4 TBL Wavenumber Spectrum

Concerning the wavenumber-frequency spectrum of TBL pressure, it is appropriate and usual (e.g., see Ref. 1) at high frequency ($\omega\delta_*/U_\infty \gg 1$, where U_∞ denotes asymptotic flow speed and δ_* the TBL displacement thickness) to distinguish (A) a high-wavenumber

convective wavenumber component ($K \approx \omega/U_\infty$) and (B) a low-wavenumber component. We may further distinguish, subject to precise definition of (B), an acoustic component (C) having $K \lesssim \omega/c$ and associated with compressibility of the flowing medium, whose sound velocity is c. This last component, however, like the radiated field (2), is not subject to relative reduction by the dome.

We assume component (B) may be represented as white noise in wavenumber. For $K \gtrsim \delta_*^{-1}$, there is some indication that this should be so (e.g. see Ref. 2). For $K \lesssim 1/3\delta_*$, this component varies rather as $(K\delta_*)^2$, but this "cut-out" in the wavenumber spectrum is filled in (though with uncertain K dependence) at $K \lesssim \omega/c$ by the component (C)(see Fig. 2). The cut-out can be significant roughly where $\omega\delta_*/c \lesssim 0.3$; in view of other uncertainties, we neglect this cut-out except as noted hereafter; its effect will tend to make the unilayer more effective in reducing noise, in some regime, than otherwise estimated.

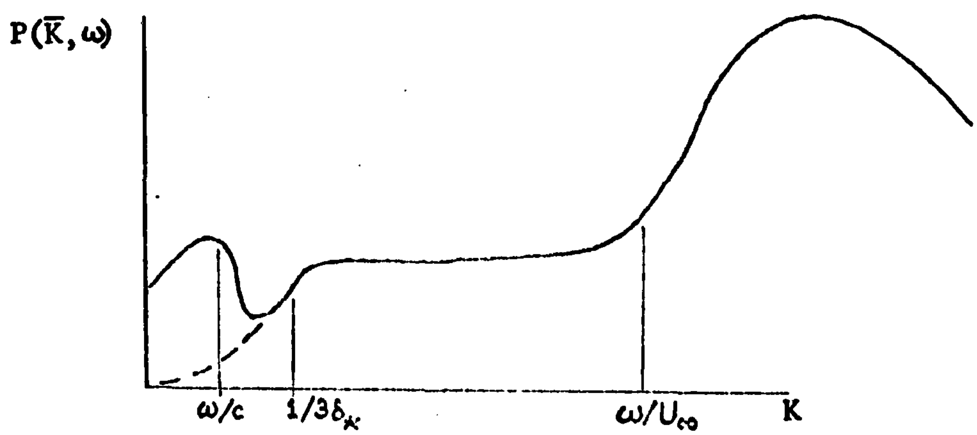

Figure 2. Schematic Dependence of Wavenumber Spectrum of TBL Pressure ($\omega\delta_*/U_\infty >> 1$).

2. Reduction of TBL Noise by a Unilayer

We compare a flush array of elements of area πR_o^2, N to a segment (see Fig. 1), with a shielded array of elements of area πr_o^2, likewise N to a segment. (We do not exclude the instance where the elements are rectangular but of comparable longitudinal and transverse extent.) We wish to compare the frequency spectra of noise pressure averaged over the entire active areas of the respective arrays, considering separately the noise components (A) and (B). We assume the segments of the unilayer are many sound wavelengths in lateral extent, i.e.

$$\omega a/c \gg \pi, \tag{1}$$

where c now refers to the sound velocity in the unilayer fluid.

2.1 High-Wavenumber Component

For a single flush element with $\omega R_o/U_\infty \gg \pi$, the spectrum of area-averaged pressure from this component varies as R_o^{-3} and is written as

$$Q_+(R_o, \omega) = (\pi R_o^2)^{-3/2} s(\omega) \tag{2}$$

(From the indicated TBL pressure scaling, in fact, we have roughly (e.g. see Ref. 3)

$$\pi^{-3/2} s(\omega) \sim \rho^2 v_*^4 U_\infty^3 \omega^{-4}, \tag{3}$$

where ρ denotes fluid density and v_* friction velocity.) Suppose first that the separation d of the edges of the active areas of streamwise adjacent elements is such that $d \gg U_\infty/\omega$.

Then the noise spectra on the nN elements of the array add incoherently, and the corresponding spectrum $Q_+^A(\omega)$ of noise pressure averaged over the entire active area $A = nN(\pi R_o^2)$ of the array is given by

$$(4) \qquad Q_+^A(\omega) = Q_+(R_o,\omega)/nN = \alpha^{-1}(\pi R_o^2)^{-1/2} A_T^{-1} s(\omega),$$

where $\alpha = A/A_T$ is the array factor and A_T is the total (active plus dead) array area. Now suppose instead that the elements within each section are rectangular with active areas so contiguous that their spacing $d \lesssim U_\infty/\omega$ so that the noise is coherently added over each array segment, but that the noise averaged over individual segments is added incoherently. Then

$$(5) \qquad Q_+^A(\omega) = Q_+(a,\omega)/n \sim (\pi a^2)^{-3/2} s(\omega)/n$$
$$\sim (\pi a^2)^{-1/2} A_T^{-1} s(\omega).$$

We turn to the high-wavenumber noise spectrum $Q_{s+}^{A'}(\omega)_N$ of pressure averaged over the active $A' = nN(\pi r_o^2)$ of the shielded array. The spectrum $Q_{s+}(\omega)$ of pressure averaged over a single shielded element of area πr_o^2 (with $\omega r_o/c \lesssim 1$) we crudely approximate as

$$\begin{array}{ll} (6a) & Q_{s+}(\omega) \\ (6b) & \end{array} \begin{cases} (R_o/a)^3 Q_+(R_o,\omega) = (\pi a^2)^{-3/2} s(\omega) \\ \exp(-2\omega L/U_\infty) Q_+(r_o,\omega) = \exp(-2\omega L/U_\infty)(\pi r_o^2)^{-3/2} s(\omega), \end{cases}$$

where the larger of the two is to be understood, i.e. the upper form applies unless the layer thickness L is so small and the

lateral dimension a so large that

$$\omega L/U_\infty \lesssim (3/2)\ln(a/r_o). \tag{7}$$

The upper form in (6) represents a rough generalized form based on a result obtained in a limiting regime corresponding to condition (1) and certain other more restrictive conditions for an approximate model of a finite fluid layer (Ref. 1). It states that the average-pressure spectrum for a shielded area (due to high wave numbers) is of the order of that for a flush averaging area equal to the entire face area of the finite fluid-dome section. This approximation must be crude and its regime of applicability at present is somewhat uncertain; for example, the true result really depends on the boundary conditions over the edge of the section. (Approximations (6) also presume the inessential restrictions of a nearly rigid inner surface and $c \gg U_\infty$.) We believe, nevertheless, that (6) is usefully indicative of the actual situation. We assume subsequently that L is large enough that form (6a) applies; this thickness is modest at the frequencies of interest.

In the limit where form (6a) is explicitly indicated to become correct, the predominant wave vectors of the interior pressure field are nearly parallel to the layer surface with the magnitude ω/c for a radiated acoustic field. (In the contrary limit corresponding to (6b), the interior spectrum of wavenumbers parallel to the surface is instead nearly the same as that of the exciting pressure.)

Since the interior noise wave vectors lie mainly in a plane parallel to the surface, we consider separately array steering angles (i) well removed from endfire and (ii) near endfire (i.e., endfire within the beamwidth established by the elements of a single segment of the array). We further distinguish the

limiting cases of (1) tight packing, i.e. interstitial distance d' between elements small relative to the interior sound wave length, say $\omega d'/c \lesssim 1$, and (2) loose packing, $\omega d'/c \gg 1$.

Letting $Q^a_{s+}(\omega)$ denote the spectrum of noise pressure averaged over the active area $N(\pi r_o^2)$ of a single segment of the shielded array, in case (i1) of non-endfire and tight packing we have Q^a_{s+} nearly equal to the spectrum of pressure averaged over the entire area πa^2 of a segment without regard to interstices. Crudely, we expect this averaging reduces the spectrum relative to a point on the shielded face by the cube of the number of noise wavelengths contained over the extent ($\sim 2a$) of the segment, or say

$$(8) \qquad Q^a_{s+}(\omega) \sim (\omega a/c)^{-3}\, Q_{s+}(\omega),$$

where $Q_{s+}(\omega)$ is estimated by (6a). Adding the noise for the independent segments incoherently, we then obtain for the average over the entire active area of the array

$$Q^{A'}_{s+}(\omega) = Q^a_{s+}(\omega)/n \sim n^{-1}(\pi a/c)^{-3}\, Q_{s+}(\omega)$$

$$(9) \qquad \sim (\omega a/c)^{-3}(\pi a^2)^{-1/2}\, A_T^{-1} s(\omega).$$

In case (i2) of loose packing the noise on individual elements adds nearly incoherently, and we obtain in place of (8) and (9)

$$Q^a_{s+}(\omega) = Q_{s+}(\omega)/N,$$

$$(10) \quad Q^{A'}_{s+}(\omega) = Q^a_{s+}(\omega)/n = (\pi a^2)^{-1/2}(D^2/\pi a^2)A_T^{-1} s(\omega)$$

where $\sigma_s \equiv A'/A_T = \pi r_o^2/D^2$ is the array factor and D is the center spacing of adjacent elements in the shielded (or flush) array.

In the endfire cases (ii1) and (ii2), on the other hand, in addition to the contribution (9) or (10) we recognize a special contribution due to interior noise wave vectors lying within the azimuthal beamwidth $\sim(\omega a/c)^{-1}$ defined by the elements within a single array segment. Specifically, this noise will add coherently with fixed phase for all elements of the segment. Hence, on assumption of noise roughly isotropic in the plane, we infer an increment $\delta Q_{s+}^{a}(\omega)$ in $Q_{s+}^{a}(\omega)$ given in order of magnitude by

$$\delta Q_{s+}^{a}(\omega) \sim (\omega a/c)^{-1} Q_{s+}(\omega), \tag{11}$$

and an increment in $Q_{s+}^{A'}(\omega)$ given by

$$\begin{aligned} \delta Q_{s+}^{A'}(\omega) &= \delta Q_{s+}^{a}(\omega)/n \\ &\sim (\omega a/c)^{-1}(\pi a^2)^{-1/2} A_T^{-1} s(\omega). \end{aligned} \tag{12}$$

Comparison of (12) with (9) for the instance of tight packing shows, in view of (1), that the special endfire part (12) is the larger. Furthermore, at the frequency of active operation of the array we have $D \simeq \lambda/2 = \pi c/\omega$, whence the increment (12), since $a \gg D$, is larger also than (10) for the instance of loose packing. Hence, so far as the high-wavenumber part of the exciting TBL spectrum is concerned, the reduction of noise by a laterally large unilayer dome is least near endfire.

In summary, we obtain the ratio σ_+ of the spectrum of noise pressure due to the high-wavenumber TBL component in the case of the shielded array to that in the case of the flush array. Various cases are involved, and the equation numbers of the estimates used to obtain the ratios are given for each.

Ia. Flush array imperfectly packed (incoherent noise addition for adjacent elements), shielded array tightly packed, array steered away from endfire:

(13) $$\sigma_+ \sim \alpha(R_o/a)(\omega a/c)^{-3}$$ (Eq.(4),(9))

IIa. Flush array imperfectly packed, shielded array loosely packed, away from endfire:

(14) $$\sigma_+ \sim \alpha(R_o/a)(D^2/\pi a^2) = (R_o/a)^3$$ (Eq.(4),(10))

IIIa. Same as Ia or IIa, but near endfire:

(15) $$\sigma_+ \sim \alpha(R_o/a)(\omega a/c)^{-1}$$ (Eq.(4),(12))

Ib. Same as Ia except flush array perfectly packed (100% array factor, no loss of coherence at interstices):

(16) $$\sigma_+ \sim (\omega a/c)^{-3}$$ (Eq.(5),(9))

IIb. Same as IIa except flush array perfectly packed:

(17) $$\sigma_+ \sim D^2/\pi a^2$$ (Eq.(5),(10))

IIIb. Same as IIIa except flush array perfectly packed:

(18) $$\sigma_+ \sim (\omega a/c)^{-1}$$ (Eq.(5),(12)).

The ratios designated by b are of questionable practical significance, since, if such perfect packing could be achieved at all in a flush array, the high-wavenumber component would likely be reduced to such a level as not to dominate the noise even on the flush array. The ratio designated by a in each case are less than the product of the array factor α of the flush comparison array and the ratio R_o/a of its element radius to the characteristic lateral dimension a of each segment of the unilayer dome.

3.2 Low-wavenumber Component

For a single flush element with $\omega R_o/U_\infty \gg \pi$ the spectrum of area-averaged pressure from this component, assumed white, varies as R_o^{-2} and is written as

$$Q\_(R_o,\omega) = 2R_o^{-2}\, I(\omega)$$

(cf. Eq.(2)). (We conjecture, in fact, that

(19) $$2I(\omega) \sim b\ \rho^2 v_*^6 \omega^{-3},$$

where the coefficient b may be substantially larger than unity.*) The noise spectra on the nN elements of the array add incoherently to yield the corresponding spectrum $Q_-^A(\omega)$ of noise pressure averaged over the active area of the array:

$$(20) \qquad Q_-^A(\omega) = Q_-(R_o,\omega)/nN = 2\pi\,\alpha^{-1}A_T^{-1}\,I(\omega).$$

With regard to the shielded array, the spectrum of pressure averaged over a single element is found (e.g., Ref. 1) to be approximated by

$$(21) \qquad Q_{s-}(r_o,\omega) \simeq \begin{cases} 2R_e^{-2}\,I(\omega) & \text{for } R_e \gtrsim r_o \\ 2\,r_o^{-2}I(\omega) & \text{for } R_e \lesssim r_o, \end{cases}$$

where the effective averaging radius R_e is given by

$$(22) \qquad R_e^{-2} = (1/4)\left[(\omega/c)^2 + (1/2)L^{-2}\right],$$

in which we may write alternatively $\omega/c = 2\pi/\lambda$.** The result (21) was derived on assumption of condition (1) and the further condition $(\omega L/c)(\omega a/c)^{-1/2} << 1$. (A rigid inner surface is still assumed.)

We distinguish the limiting cases of tight and loose

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*If we accept estimates (3) and (19), we have for the ratio of low- to high-wavenumber TBL noise on a flush element

$$(19.1) \qquad Q_-(R_o,\omega)/Q_+(R_o,\omega) \sim b(v_*/U_\infty)^2(\omega R_o/U_\infty).$$

The low-wavenumber part then represents a small but increasing fraction of the TBL noise through the frequency range of interest.

**The area scale of the noise-pressure field at the inner surface is just πR_e^2.

packing, which in the context of the low-wavenumber component are defined, at given ω, roughly by $d' \ll R_e$ and $d' \gtrsim R_c$, respectively. In the former case of tight packing, the spectrum $Q_{s-}^{a}(\omega)$ of noise pressure averaged over the active area of a segment is nearly equal to the spectrum of pressure average over the entire area πa^2 without regard to interstices. For the assumed white spectrum, we have (since $a \gg R_e$)

$$(23) \qquad Q_{s-}^{a}(\omega) \propto 2a^{-2} I(\omega).$$

Adding the noise for the independent segments incoherently, we then obtain for the entire array

$$(24) \qquad Q_{sa-}^{A'}(\omega) = Q_{s-}^{a}(\omega)/n \simeq 2\pi A_T^{-1} I(\omega).$$

In the case of loose packing we have simply

$$(25) \qquad Q_{s-}^{A'}(\omega) = Q_{s-}(r_o,\omega)/nN = D^2 A_T^{-1} Q_{s-}(r_o,\omega),$$

with Q_{s-} given by (21).

In summary, on the basis of (20)-(25), we obtain the ratio σ_- of the spectrum of noise due to the low-wavenumber component for the shielded array to that for the flush array (Ref. 1)*:

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*If the elements of the flush reference array are taken equivalent to those of the shielded array, we note, form (26a) reduces to unity.

$$\sigma_- \sim \begin{cases} (R_o/r_o)^2 \text{ for } R_e \lesssim r_o & (26a) \\ \propto (\Pi^2/\pi\, R_e^2) = (R_o/R_e)^2 \text{ for } r_o \lesssim R_e \lesssim D & (26b) \\ \alpha = \pi R_o^2/D^2 \text{ for } D \lesssim R_e & (26c) \end{cases}$$

In the domain where $R_e >> \delta_*$, (26b) and (26c) should be reduced by a further factor $\sim 6\delta_*/R_e$ to take account of the cutout at $K \leq 1/3\delta_*$ in Fig. 2.

According to (26), the unilayer reduces the low-wavenumber noise at a given frequency (recall $R_e = R_e(\omega)$) only if the thickness satisfies

$$L \gtrsim 0.36\, r_o(1-\pi^2 r_o^2/\lambda^2)^{-1/2}, \quad (27)$$

and reduces this noise by the maximum attainable factor (α^{-1}) at all frequencies less than ω ($=2\pi c/\lambda$) provided L satisfies the more stringent condition

$$L \gtrsim 0.2D(1-\pi D^2/\lambda^2)^{-1/2}. \quad (28)$$

If $D \simeq \lambda/2$, the latter condition becomes

$$L \gtrsim 0.43D. \quad (29)$$

Comparing estimates (13)-(15) with (26), we see that, relative to an imperfectly packed flush array, the unilayer reduces the high-wavenumber noise more than the low-wavenumber noise by a factor greater than a/R_o ($>>1$). Hence, even in the absence of acoustic noise excitation, in order to assess the noise reduction by the layer we must know the relative contributions from high and low wavenumbers to the noise on a flush element. Measurements (e.g. Ref. 3) suggest that up to the

largest values of $\omega R_o/U_\infty$ of concern the high-wavenumber component is at least comparable with the low. Likewise, if we accept the estimated ratio (19.1) and suppose $b \sim 10$, for example, at $\omega/2\pi$ = 2 kh with R_o = 2.5" and U_∞ = 15 kt we find the ratio is unity. In the absence of acoustic noise excitation, then, the factor of noise reduction may lie somewhere between a value somewhat larger than the reciprocal array factor (α^{-1}) up to a factor larger than $\alpha^{-1}a/R_o$, e.g. a reduction > 10 db + 10 log α^{-1} if $a/R_o = 10$.

If radiated acoustic noise (including that related to compressibility of the TBL) predominates on a flush element in the configuration and frequency range of interest, however, no such gain from a dome can be realized. An additional complication is the pressure field associated with resonant bending waves in the hull surrounding a flush-mounted element; this contribution to noise will be reduced in some degree by a superposed unilayer dome, or otherwise affected by a covered dome capable itself of sustaining resonant waves parallel to its surface.

A reliable prediction of the noise reduction by a dome thus depends essentially on identification of the character of the predominant noise source in the frequency range of interest in previous sea trials and thence of the dominant source in suitably altered configurations.

3. Extension to Realizable Domes

One type of realizable dome preserves the unilayer character considered here but is composed not by a fluid but by an elastic solid. In this instance we believe that the estimates above remain crudely applicable provided the sound velocity c in the unilayer assumed here is identified as the shear sound velocity c_t in the elastic material and that $c_t \gg U_\infty$. The validity of this rough assumption with reference to an infinite solid unilayer and high-wavenumber excitation

was shown explicitly in Ref. 4.

Another type of realizable dome consists of two layers, the inner being fluid but the outer again an elastic solid, both subject to perhaps complex boundary conditions at the periphery of the layer segments. If the acoustic impedance of the solid as a function of frequency and wavenumber differs only moderately from that of a similar thickness of the fluid, the present estimates again are expected to be crudely applicable.

If, on the other hand, the solid cover is steel or some similar material, the effect of the dome on the noise components distinguished here is somewhat altered, but, more crucially, the effect on noise may become determined primarily by the interior acoustic field associated with resonant waves (e.g. bending waves) in the fluid-loaded dome cover. Much useful effort has been devoted to the study of such structures (e.g. Refs. 5,6,1), but quantitative predictions at present appear fraught with uncertainty and we shall not attempt such here. The pressure field associated with resonant waves will be transmitted to the array directly via the interior fluid only with great attenuation, provided the resonant wave number k_r satisfies $k_r \gg \omega/c$ and $k_r L \gtrsim 1$ (cf. (6b)); indirect excitation via the lateral or supporting dome structure, however, may also occur.

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